Simulation of Proton Acceleration in a Oscillating Magnetic Island in the Folds of the Heliospheric Current Sheet

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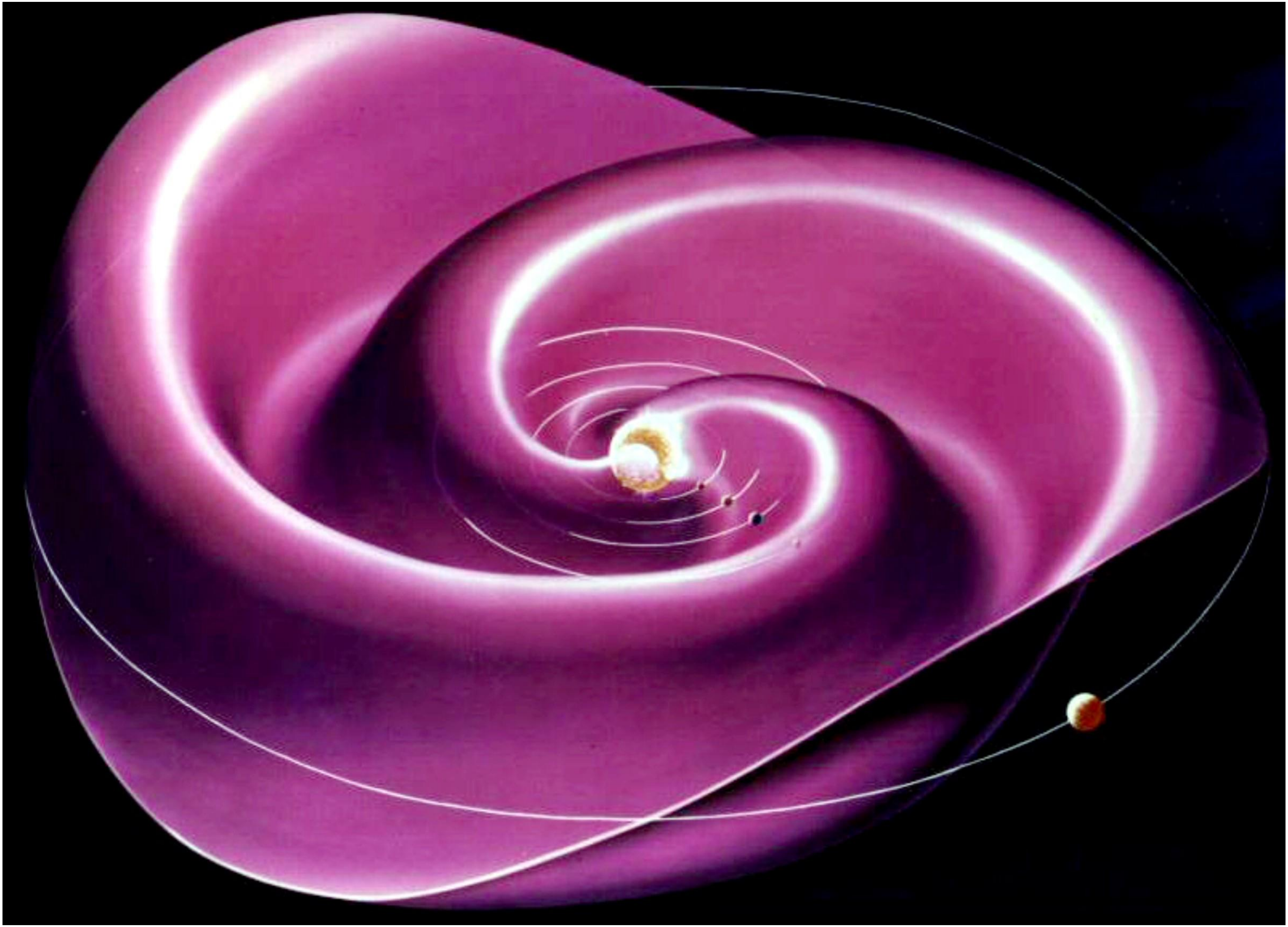

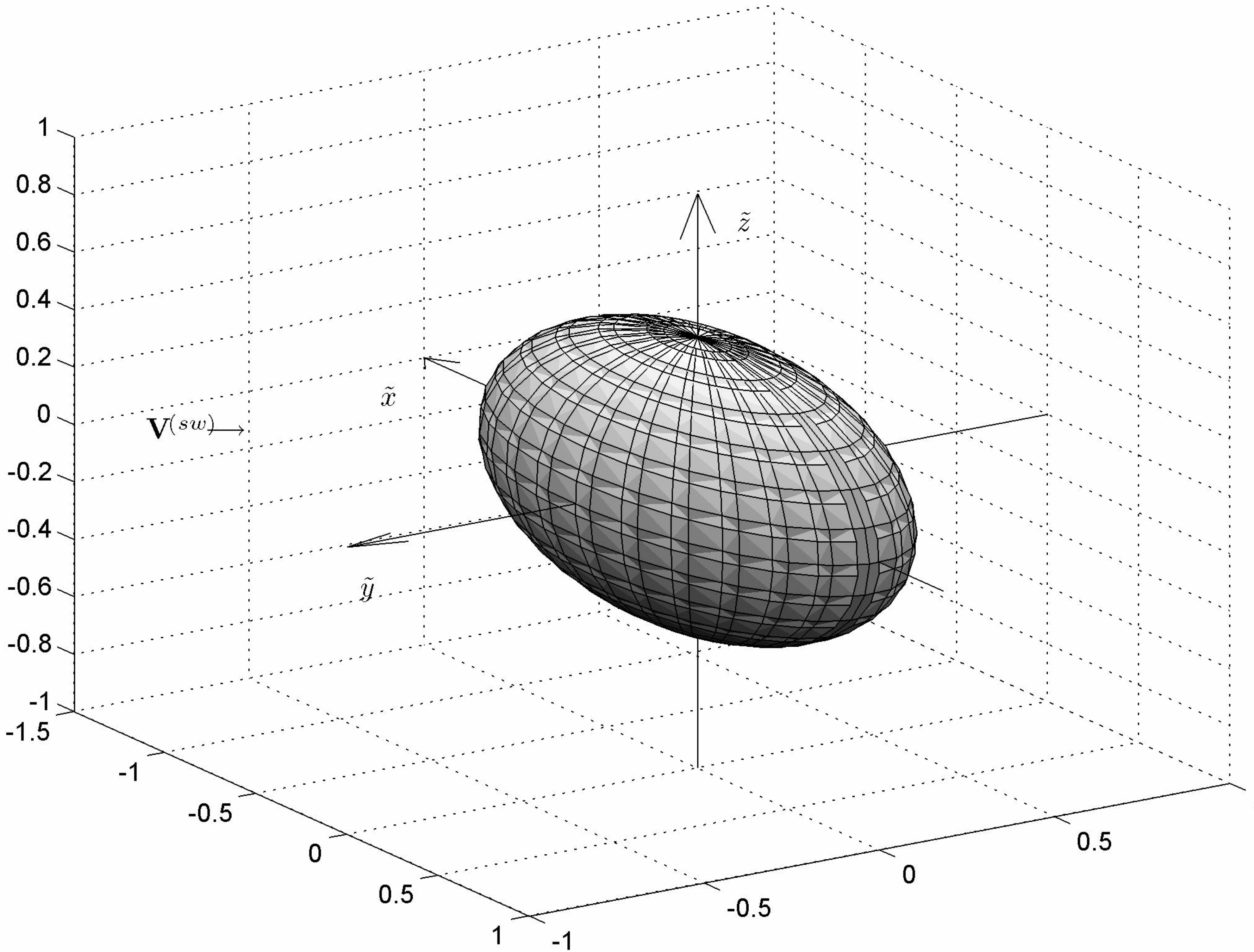

1
0.8
0.6
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-0.2
-0.4
-0.6
-0.8
-1
-1.5
-1
-0.5
0
0.5
1
-1
-0.5
0
0.5
1
$\tilde{z}$
$\tilde{x}$
$\tilde{y}$
$\mathbf{V}^{(sw)}$

Objectives

- Make a analytical model of electromagnetic field for a electrically neutral oscillating magnetic island
- Calculate trajectories of protons up to emit from island with different parameters
- Make a conclusions about acceleration in magnetic island

System`s geometry

We assume inside the folds background fields of solar wind is homogeneous and stationary and in GSM coordinate system have form:

$$\boldsymbol{B}^{(sw)} = B_{x0}\,\boldsymbol{e}_x + B_{y0}\,\boldsymbol{e}_y\,, \quad \boldsymbol{E}^{(sw)} = E_{z0}\boldsymbol{e}_z\,.$$

Then the solar wind speed has form:

$$\boldsymbol{V}^{(sw)} = \boldsymbol{v}_E = \frac{\left[\boldsymbol{E}^{(sw)} \times \boldsymbol{B}^{(sw)}\right]}{\left|\boldsymbol{B}^{(sw)}\right|^2} = \frac{E_{z0}\left(-B_{y0}\,\boldsymbol{e}_x + B_{x0}\,\boldsymbol{e}_y\right)}{\left|B_{x0}\right|^2 + \left|B_{y0}\right|^2}\,.$$

Next, we use K` coordinate system which has an axis parallel to the axis of GSM, and moves together with solar wind with the speed $\boldsymbol{V}^{(sw)}$. And in this coordinate system E and B look like:

$$\boldsymbol{E}^{(sw)\prime} = 0 \qquad \boldsymbol{B}^{(sw)\prime} = \frac{1}{\sqrt{1-\left(V^{(sw)}/c\right)^2}}\left(1-\left(\left|\boldsymbol{E}^{(sw)}\right|/\left(c\left|\boldsymbol{B}^{(sw)}\right|\right)\right)^2\right)\boldsymbol{B}^{(sw)} \approx \boldsymbol{B}^{(sw)}$$

Further, we consider magnetic island moving along with the solar wind in the K` system, in which center of island is stationary, and full filds have form:

$$\boldsymbol{B}(\boldsymbol{x},t) = \boldsymbol{B}^{(sw)} + \boldsymbol{B}^{(i)}(\boldsymbol{x},t)\,, \quad \boldsymbol{E}(\boldsymbol{x},t) = \boldsymbol{E}^{(i)}(\boldsymbol{x},t)$$

Model of fields in the magnetic island

A magnetic island is assumed to be electrically neutral, i.e. the fields of currents flowing in it are determined by the vector potential by formulas:

$$\boldsymbol{B}^{(i)}(\boldsymbol{x},t) = \mathrm{rot}\,\boldsymbol{A}(\boldsymbol{x},t), \quad \boldsymbol{E}^{(i)}(\boldsymbol{x},t) = -\frac{\partial \boldsymbol{A}(\boldsymbol{x},t)}{\partial t}$$

To automatically fulfill the condition of electroneutrality, it is convenient to specify the vector potential in the form:

$$\boldsymbol{A}(\boldsymbol{x},t) = LB_m \mathrm{rot}\left(\psi_1(\boldsymbol{x},t)\nabla\psi_2(\boldsymbol{x})\right) = LB_m\left[\nabla\psi_1(\boldsymbol{x},t)\times\nabla\psi_2(\boldsymbol{x})\right]$$

From the formulas given above, the following expressions for the fields follow through the functions of the Euler potentials:

$$\left.\begin{aligned} \boldsymbol{B}^{(i)}(\boldsymbol{x},t) &= \mathrm{rot}\,\boldsymbol{A}(\boldsymbol{x},t) = LB_m \mathrm{rot}\,\mathrm{rot}\left(\psi_1(\boldsymbol{x},t)\nabla\psi_2(\boldsymbol{x})\right), \\ \boldsymbol{E}^{(i)}(\boldsymbol{x},t) &= -\frac{\partial \boldsymbol{A}(\boldsymbol{x},t)}{\partial t} = LB_m\left[\nabla\psi_2(\boldsymbol{x})\times\nabla\frac{\partial\psi_1(\boldsymbol{x},t)}{\partial t}\right]. \end{aligned}\right\}$$

where $L = L_x$: $L = 10^8;\ 1.5\cdot10^8;\ 2\cdot10^8$ м $\quad L_y = L_z = L/2$ and $B_{x0} = B_{y0} = 5$ нТл

$B_m = \frac{1}{2}B_{x0};\ \frac{3}{4}B_{x0};\ B_{x0}$

Method of calculation proton trajectories

To calculate the particle trajectory we have to solve numerically the Cauchy problem for the system of Newton-Lorentz equations, which in the SI in the relativistic case has form:

$$\frac{d\boldsymbol{x}(t)}{dt}=\boldsymbol{v}(t),\ \ \boldsymbol{p}(\boldsymbol{v})=\frac{m_0\boldsymbol{v}}{\sqrt{1-|\boldsymbol{v}|^2/c^2}},\ \ \frac{d\boldsymbol{p}(\boldsymbol{v}(t))}{dt}=e\Big(\boldsymbol{E}(\boldsymbol{x}(t),t)+\big[\boldsymbol{v}(t)\times\boldsymbol{B}(\boldsymbol{x}(t),t)\big]\Big),$$

In classical mechanics *|**v**|/c<<1* this system take form:

$$\frac{d\boldsymbol{x}(t)}{dt}=\boldsymbol{v}(t),\quad \frac{d\boldsymbol{v}(t)}{dt}=\frac{e}{m_0}\Big(\boldsymbol{E}(\boldsymbol{x}(t),t)+\big[\boldsymbol{v}(t)\times\boldsymbol{B}(\boldsymbol{x}(t),t)\big]\Big)$$

The kinetic energy in electron-volts is related to the velocity modulus:

$$W(|\boldsymbol{v}|)=\frac{m_0c^2}{e}\left(\frac{1}{\sqrt{1-|\boldsymbol{v}|^2/c^2}}-1\right),\qquad \frac{|\boldsymbol{v}|}{c}=\sqrt{\frac{eW}{m_0c^2}\left(\frac{eW}{m_0c^2}+2\right)}\Bigg/\left(\frac{eW}{m_0c^2}+1\right)$$

And in classical mechanics it transform to: $W(v)\rightarrow\frac{1}{2e}m_0v^2$ when $\frac{v}{c}\rightarrow 0$

For economy calculation resourse when $\beta=|\boldsymbol{v}|/c<\beta_0=0.02$ use equatıon of classical mechancs, when $\beta\geq\beta_0=0.02$ use equation of relativistic case

Method of modeling

We have chosen a sufficiently wide set of initial points $\{\boldsymbol{x}^0[k_x]\}$ inside the magnetic island and set of initial instants of time corresponding to different phases of island`s oscillations too

$$\left\{t^0[k_t]=0, \frac{\Theta}{16}, \frac{\Theta}{8}, \frac{3\cdot\Theta}{16}, \frac{\Theta}{4}, \frac{5\cdot\Theta}{16}, \frac{3\cdot\Theta}{8}, \frac{7\cdot\Theta}{16}, \frac{\Theta}{2}, \frac{3\cdot\Theta}{4}, \frac{7\cdot\Theta}{8}\right\}$$

We have also chosen a set of kinetic energies $W^0[k_w]$ with range from 10 eV to 100 keV, with step is 10 eV for range from 10eV to 0.5 keV and with step is 0.5keV for range from 0.5keV to 100 keV.

Each initial energy corresponds to the initial velocity

$$v^0\left(W^0\right)=c\sqrt{\frac{eW^0}{m_0c^2}\left(\frac{eW^0}{m_0c^2}+2\right)}\Bigg/\left(\frac{eW^0}{m_0c^2}+1\right)$$

Each initial velocity has a distribution on the angle of the direction in steps of 1 degree as well

$$\left\{\boldsymbol{v}^0\left[W^0,k_\alpha,k_\beta\right],\ k_\alpha=-N_\alpha+1,\ldots,N_\alpha,\ k_\beta=0,\ldots,N_\beta,\ N_\alpha=N_\beta=180\right\}$$

Method of modeling

The trajectory control area is a rectangular parallelepiped

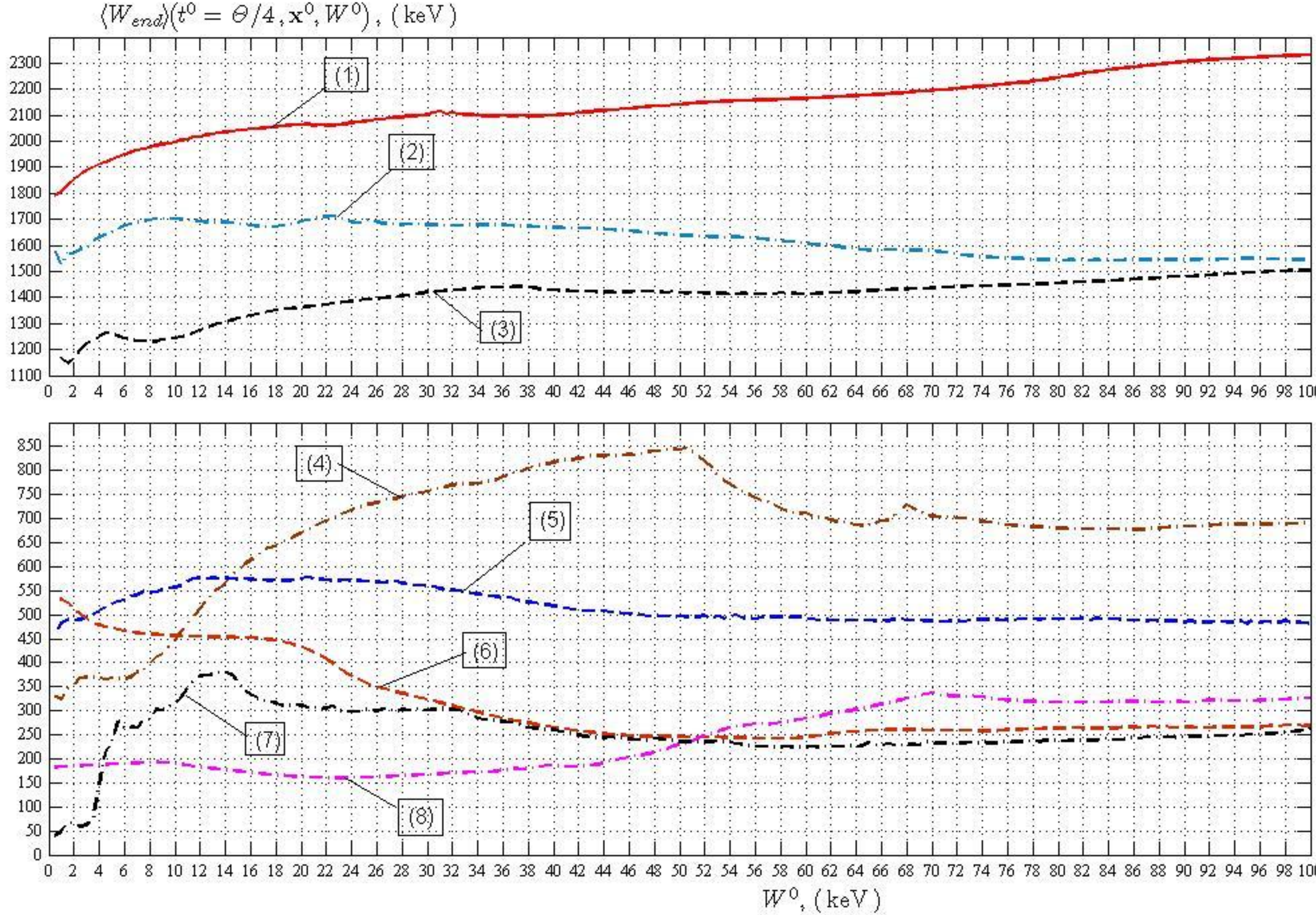
$$\Pi = \left\{ \boldsymbol{x} : \ \left| x/L \right| \le 1.25, \ \left| y/L \right| \le 1, \ \left| z/L \right| \le 0.75 \right\}$$

that contains an ellipsoid — magnetic island

$$\Omega_0 = \left\{ \boldsymbol{x} : \ \left(x^2 + 4y^2 + 4z^2 \right) / L^2 \le 1 \right\}.$$

As a result, the time $T_{end}\left(t^0, \boldsymbol{x}^0, \boldsymbol{v}^0\right)$ that the trajectory was in the control area Π is calculated, as well as the coordinate of the departure point $\boldsymbol{x}_{end}\left(t^0, \boldsymbol{x}^0, \boldsymbol{v}^0\right)$ and the departure speed $\boldsymbol{v}_{end}\left(t^0, \boldsymbol{x}^0, \boldsymbol{v}^0\right)$ from which you can calculate the energy of departure $W_{end}\left(t^0, \boldsymbol{x}^0, \boldsymbol{v}^0\right)$

It is convenient to enter the average, maximum and minimum energy of proton and the average, maximum and minumum time. Also of interest is the anisotropy of protons emitted from the control region Π. It can be described by the distribution function of the flight direction of the velocity at emission angles in spherical coordinates



$\langle W_{end}\rangle(t^0 = \Theta/4, \mathbf{x}^0, W^0)$, (keV)
(1)
(2)
(3)
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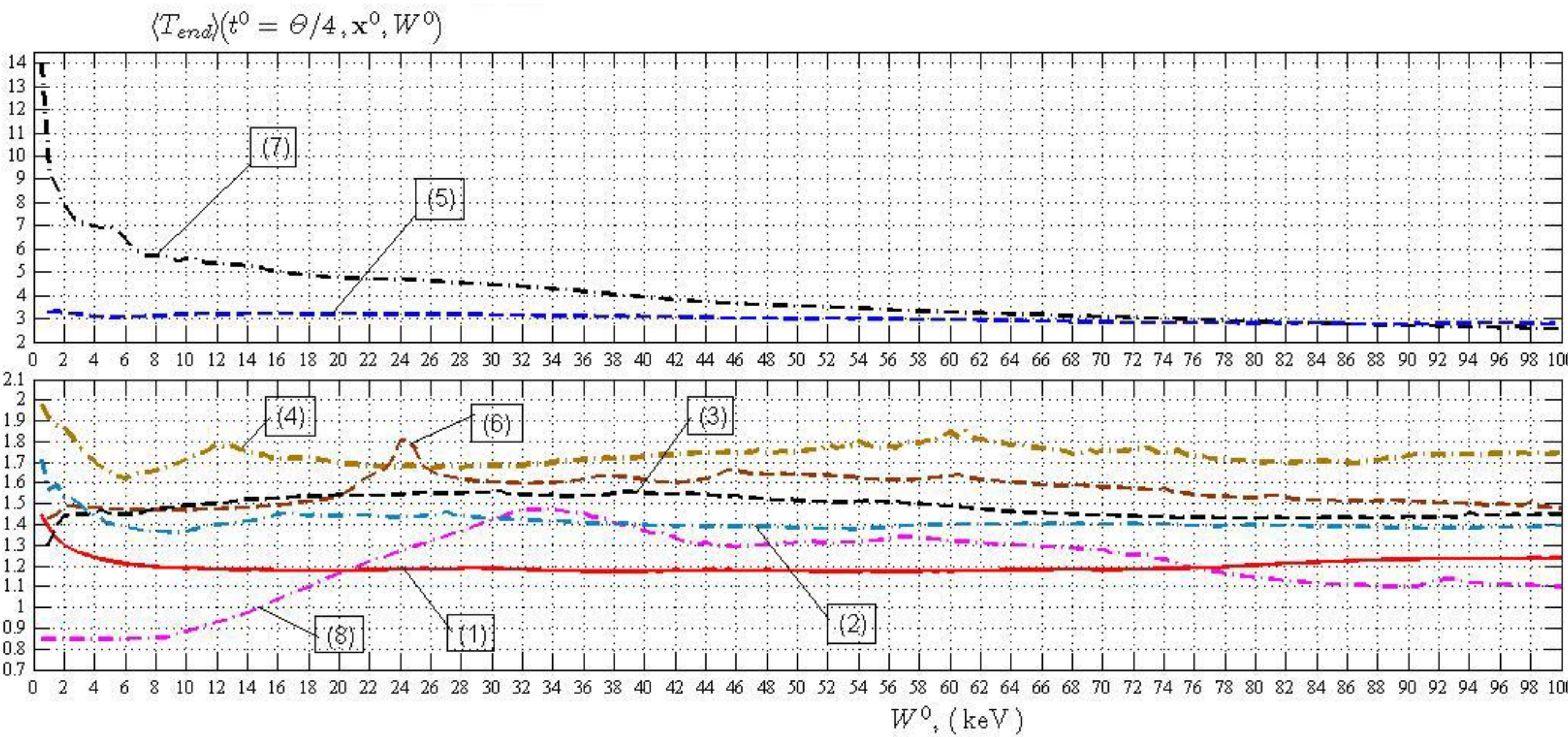
(6)
(7)
(8)
W^0, (keV)


$\langle T_{end}\rangle(t^0 = \Theta/4, \mathbf{x}^0, W^0)$
(7)
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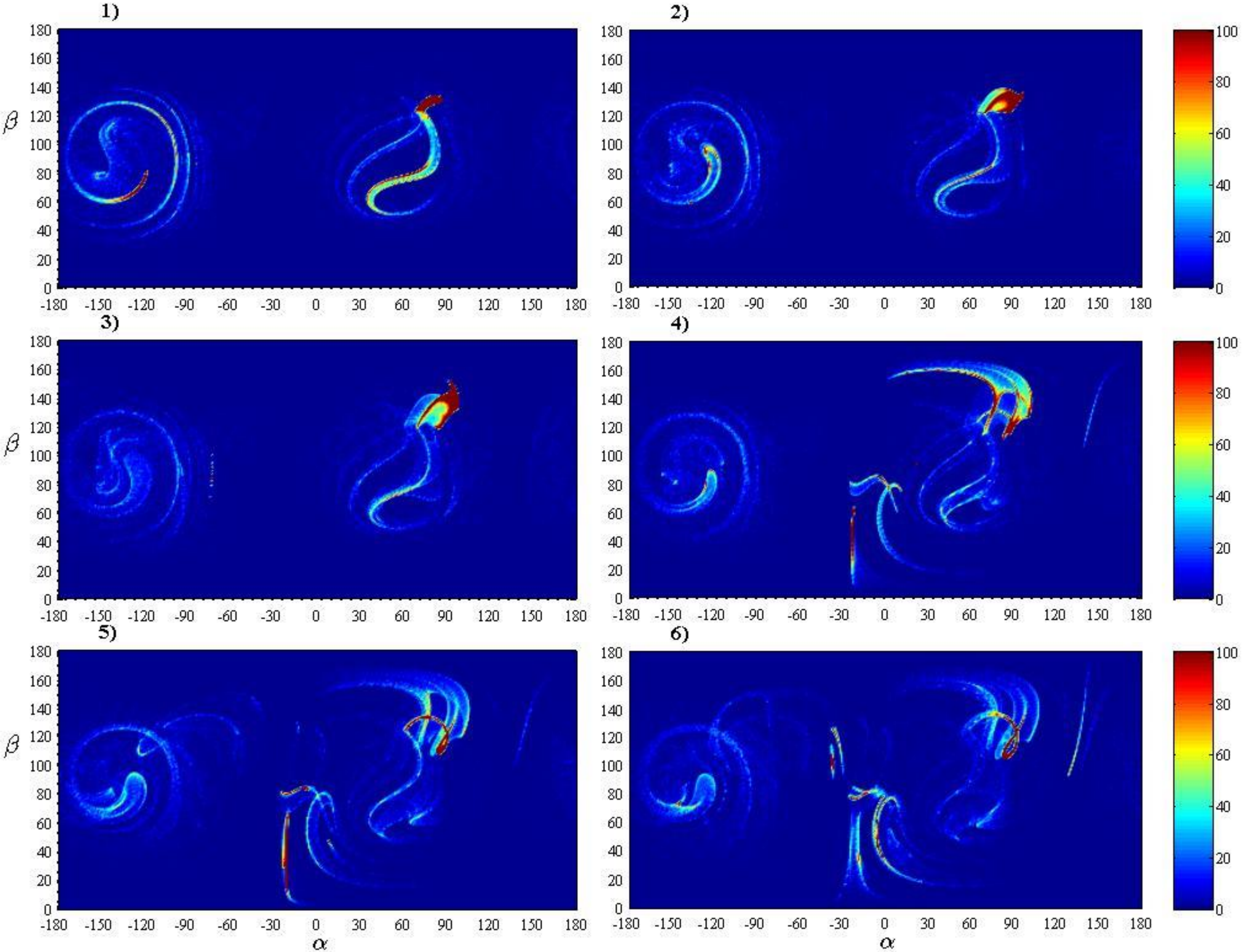
(8)
(1)
(2)
W^0, (keV)


1)
2)
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4)
5)
6)
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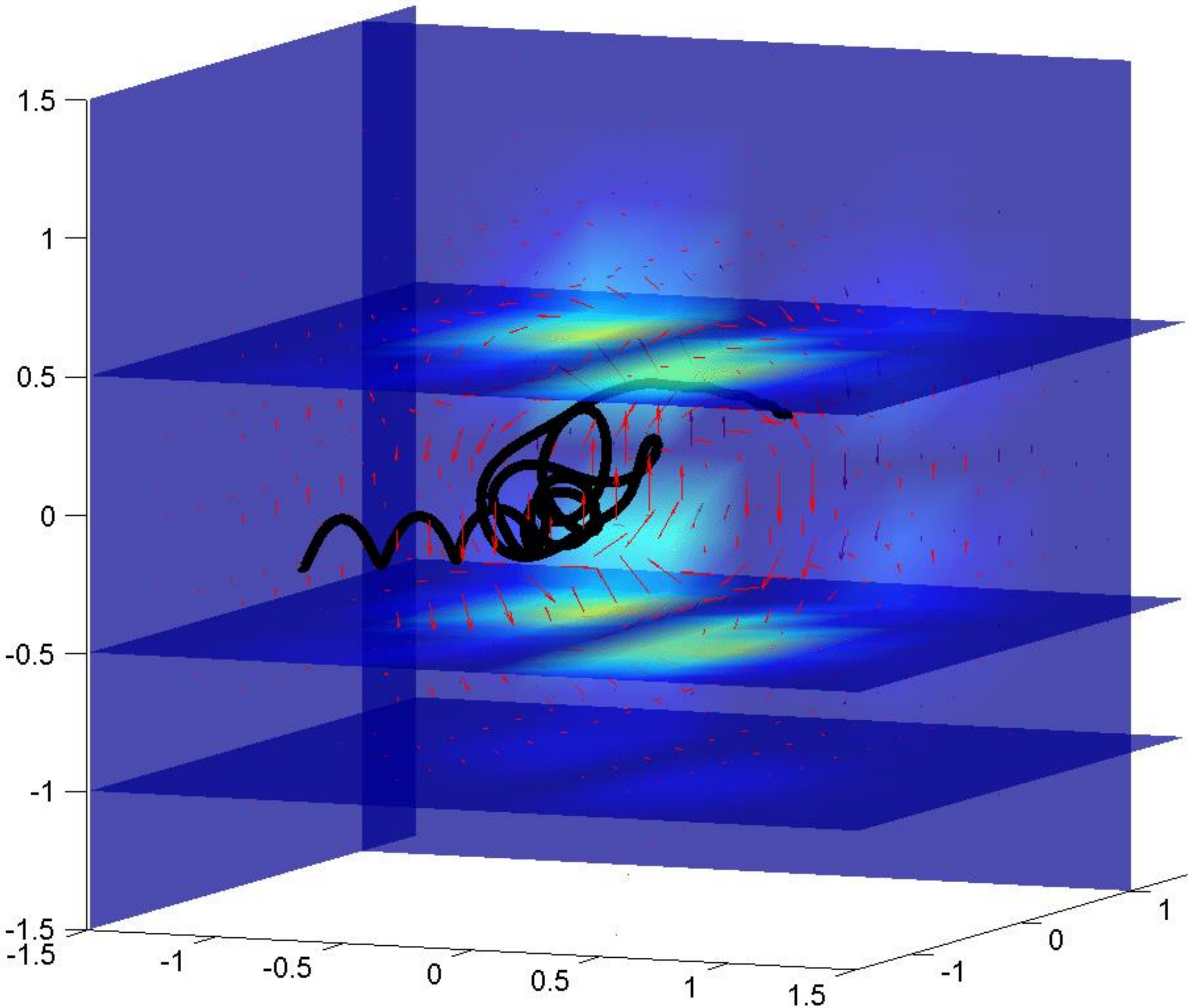
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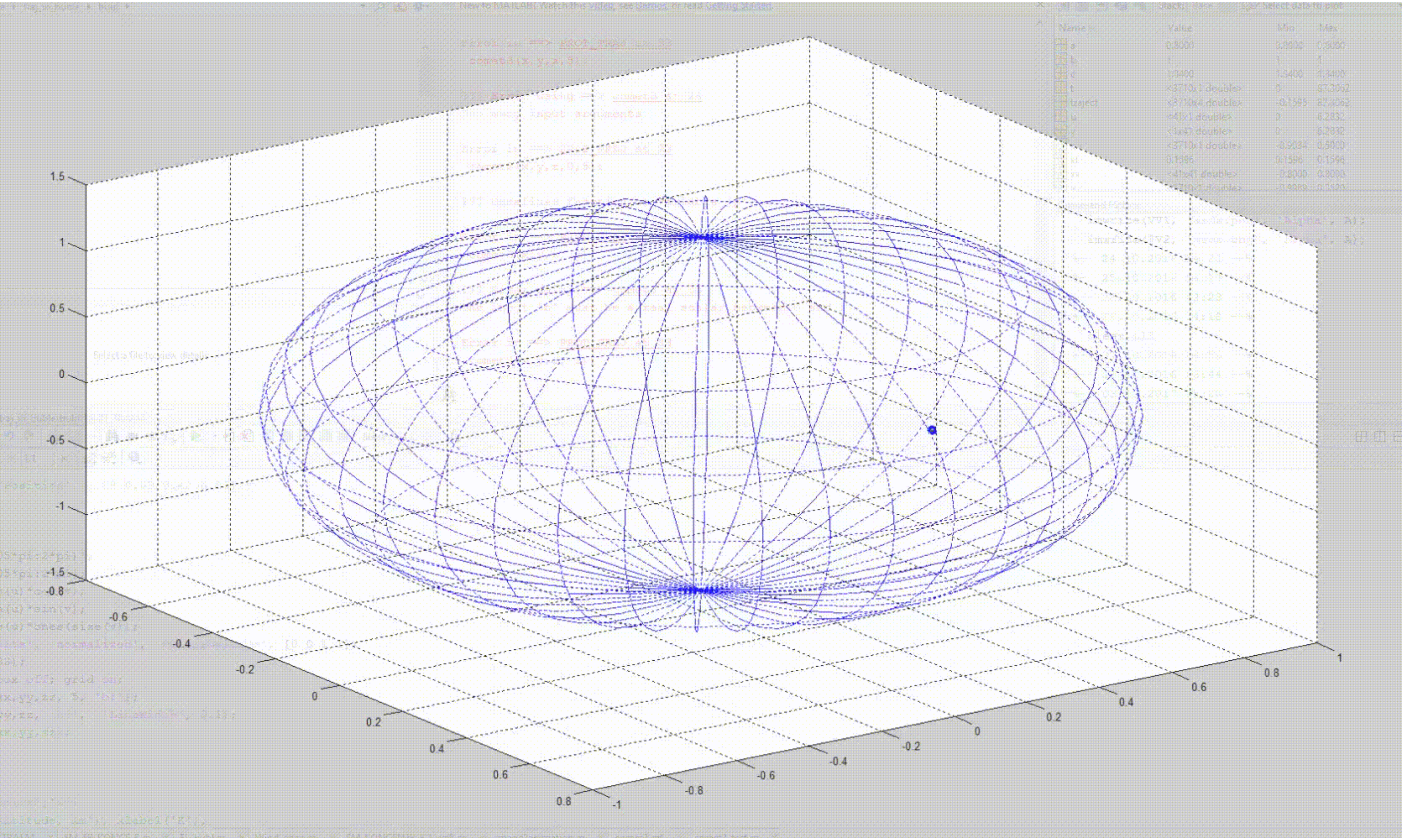
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0.6
0.8
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-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Conclusions

- The simulation results show that the model demonstrates a diverse and complex behavior, which changes abruptly with small changes in both the scenario and the input parameters.
- The model is well agreed with the measurement data for proton fluxes
- Thus, an oscillating magnetic island in the solar wind can be an effective proton accelerator

Thanks for your attention!